

MATHEMATISCH CENTRUM

2e BOERHAAVESTRAAT 49

AMSTERDAM

REKENAFDELING

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Computation of the Ginzl-Ludwig correction factor K for a
three-bladed propeller in a homogenous flow.

by

The Staff of the Computation Department

Report R 195 B

1 9 5 6

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1. On behalf of the Netherlands Ship Model Basin the Ginzell-Ludwig correction factor K [1] was computed for the values

$$\sigma = 0.2, 0.5, 0.7, 0.9$$

$$\lambda = 0.1, 0.2, 0.25, 0.4, 1$$

with a profile defined by $1/D = 1.66628 \sqrt{1-x} - 1.63267(1-x)$
In the computations the values of the Goldstein factor $\chi^{(3)}(x)$, given in report R 195 A [2] were used.

2. Following Ginzell-Ludwig [1] the following formulae were used:

$$\bar{x} = (x^2 + \lambda^2)^{\frac{1}{2}}$$

$$\varphi(x) = 2^{-\frac{1}{2}} \bar{x}^{-1} 1/D$$

$$\bar{\varphi} = \varphi(2^{-\frac{1}{2}})$$

$$\Gamma_0(x) = 2^{-1} \pi (x/\bar{x})^2 \chi^{(3)}(x)$$

$$\bar{\Gamma}_0 = 2 \int_0^1 x \Gamma_0(x) dx$$

$$\bar{\sigma}(x) = 1 + (\sigma - x) \bar{x}^{-1}$$

$$k^2(x) = 4 \bar{\sigma}(x) \{1 + \bar{\sigma}(x)\}^{-2}$$

$$f(x) = x \varphi(x)$$

$$g(x) = \lambda \varphi(x)$$

$$H(x) = \Gamma_0(x) \varphi^{-1}(x)$$

$$\alpha = \frac{\pi}{3} + \bar{\varphi}$$

$$\beta = \frac{\pi}{3} - \bar{\varphi}$$

$$I_1(\sigma, \lambda) = \frac{1}{4\pi} \int_0^1 H(x) \frac{\{\sigma f(x) + \lambda g(x)\} - (x - \sigma) \{\sigma f'(x) + \lambda g'(x)\}}{(\lambda^2 + \sigma^2)^{\frac{1}{2}} \{(x - \sigma)^2 + f^2(x) + g^2(x)\}^{\frac{1}{2}}} dx$$

$$I_2(\sigma, \lambda) = - \frac{1}{4\pi} \int_0^1 H'(x) \frac{x\sigma + \lambda^2}{\pi^2 (\lambda^2 + \sigma^2)^{\frac{1}{2}}} \left[\frac{1}{1 - \bar{\sigma}(x)} \left\{ E(k) - E\left(k, \frac{\pi - \varphi(x)}{2}\right) \right\} + \right. \\ \left. - \frac{k^2 \sin \varphi(x)}{2(1 - k^2 \cos^2 \frac{\varphi(x)}{2})^{\frac{1}{2}}} \right] + \frac{1}{1 + \bar{\sigma}(x)} \left\{ K(k) - F\left(k, \frac{\pi - \varphi(x)}{2}\right) \right\} dx$$

$$I_3(\sigma, \lambda) = \frac{\bar{\Gamma}_0}{4\pi \bar{\varphi} (\sigma^2 + \lambda^2)^{\frac{1}{2}}} \sum_{i=1}^4 C_i$$

$$C_1 = - \frac{\lambda^2 \bar{\varphi} \cos \alpha + \sigma^3 \sin \alpha}{\lambda^2 \bar{\varphi}^2 + \sigma^2 \sin^2 \alpha} \left\{ \frac{1 + \sigma \cos \alpha}{(\sigma^2 + 2 \sigma \cos \alpha + 1 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} - \frac{\sigma \cos \alpha}{(\sigma^2 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} \right\}$$

$$C_2 = \frac{\sigma^3 \sin \beta - \lambda^2 \bar{\varphi} \cos \beta}{\lambda^2 \bar{\varphi}^2 + \sigma^2 \sin^2 \beta} \left\{ \frac{1 + \sigma \cos \beta}{(\sigma^2 + 2 \sigma \cos \beta + 1 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} - \frac{\sigma \cos \beta}{(\sigma^2 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}} \right\}$$

$$C_3 = \frac{-\lambda^2(\sigma + \frac{1}{2}) + \sigma(1 + \frac{1}{2}\sigma)}{\lambda^2(\sigma^2 + \sigma + 1) + (1 + \frac{1}{2}\sigma)^2} \left\{ \frac{\frac{1}{2} \cdot 3^{\frac{1}{2}} \sigma + \bar{\varphi}(1 + \lambda^2)}{[\sigma^2 + \sigma + 1 + \bar{\varphi}^2(1 + \lambda^2) - 3^{\frac{1}{2}} \sigma \bar{\varphi}]^{\frac{1}{2}}} + \right. \\ \left. - \frac{\frac{1}{2} \cdot 3^{\frac{1}{2}} \sigma - \bar{\varphi}(1 + \lambda^2)}{[\sigma^2 + \sigma + 1 + \bar{\varphi}^2(1 + \lambda^2) + 3^{\frac{1}{2}} \sigma \bar{\varphi}]^{\frac{1}{2}}} \right\}$$

$$C_4 = \frac{2 \lambda^2 \bar{\varphi}}{\sigma(\sigma^2 + \lambda^2 \bar{\varphi}^2)^{\frac{1}{2}}}$$

where primes denote differentiation with respect to x .

3. Results

	$\lambda \rightarrow$	0.1	0.2	0.25	0.4	1
I_1	$\sigma \downarrow$					
	0.2	2.6067	2.1293	1.9654	1.5920	.8406
	0.5	2.3414	1.9831	1.8222	1.4236	.6998
	0.7	2.7747	2.2382	2.0287	1.5423	.7220
	0.9	5.4409	3.9466	3.4903	2.5603	1.1496
I_2	0.2	-.1568	-.2062	-.1576	-.0093	+.1034
	0.5	-.3789	-.0532	+.0237	+.1056	+.1054
	0.7	-.4530	+.1605	+.2471	+.2713	+.1391
	0.9	+.8098	+.8207	+.7243	+.5112	+.2036
I_3	0.2	-.3012	+.1581	+.3123	+.5085	+.3798
	0.5	-.1120	-.0577	-.0320	+.0238	+.0596
	0.7	-.0462	-.0277	-.0186	+.0027	+.0238
	0.9	-.0115	-.0066	-.0041	+.0021	+.0106

	$\lambda \rightarrow$	0.1	0.2	0.25	0.4	1
I	$\sigma \downarrow$ 0.2	2.1487	2.0812	2.1201	2.0912	1.3238
	0.5	1.8504	1.8722	1.8139	1.5530	.8648
	0.7	2.2754	2.3710	2.2572	1.8163	.8849
	0.9	6.2392	4.7607	4.2105	3.0736	1.3638
K	0.2	1.198	1.021	.929	.722	.658
	0.5	1.009	.911	.878	.818	.731
	0.7	.844	.724	.703	.677	.654
	0.9	.460	.463	.467	.472	.478

4. References

- [1] I. GINZEL, H. LUDWIG: Zur Theorie der Breitblattschraube. Deutsche Luftfahrtforschung: Untersuchungen und Mitteilungen Nr. 3097, (1944).
- [2] Staff of the Computation Department of the Mathematical Centre, Amsterdam.
R 195 A: Computation of the Goldstein factor κ for a three-bladed propeller in a homogenous flow (1956).